EE6412 - Optimal Control

Final project report

Micro-grid Energy Management

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1 Understanding of the problem

1.1 Microgrid Description

Micro-grids can be defined as 'Electricity distribution systems containing loads and distributed energy resources, (such as distributed generators, storage devices, or controllable loads) that can be operated in a controlled, coordinated way either while connected to the main power network or while islanded.' The objective of the problem is to reduce the cost associated with running the micro-grid, while meeting the power demand.

The model presented by Heymann et al [2] is used as a base reference.The micro-grid consists of a number of units, like power generation unit, power storing unit, energy management unit, and power consuming unit. For the problem addressed in this project, power is being generated by a photo-voltaic power plant, a diesel generator and an external power source (EPS) (similar to a macro-grid capable of providing continuous power supply). A battery energy storage system (BESS) is used to store excess power being produced for later use. An energy management system coordinates between all these units.

The cost of running the micro-grid is the sum of cost of diesel being consumed, startup cost for the generator and cost of importing power from the EPS. We assume the unit cost of importing power from the EPS is constant and also a negative cost is associated, if power is being exported to EPS. Cost associated with photo-voltaic power plant is assumed to be zero. Also we assume that there are no transmission losses in the grid. The solar power being generated cannot be controlled and is an independent variable. The diesel generator has minimum and maximum power that it can produce. The BESS has a limited capacity and power.

1.2 Problem Definition

The formulation of the optimal control problem builds upon the problem presented by Heymann et al [2].The horizon is fixed at $T = 48$ hours. For $t \in [0, T]$, , we denote the power supplied by the solar panels as $P_S(t)$, the power generated by the diesel generator as $P_D(t)$, the electricity load as $P_L(t)$, the power drawn from the EPS as $P_E(t)$. The state of charge SOC(t) of the BESS is regulated by the following equation:

$$\dot{SOC}(t) = \frac{1}{Q_B}(P_I(t)\rho_I - \frac{P_O(t)}{\rho_O}) \tag{1}$$

where Q_B is the maximum capacity of the battery, $P_I, P_O > 0$ are the input and output power of the BESS with corresponding efficiency ratios $\rho_I, \rho_O \in [0, 1]$.

The power equilibrium is given by:

$$P_D + P_O + P_S + P_E - P_L - P_I + P_{slack} = 0 \tag{2}$$

The cost function is given by:

$$\int_0^T (KP_D(t)^{0.9} + c_E P_E(t))dt \tag{3}$$

with K obtained from [1]. Additional constraints are as follows:

$$P_O(t, P_D) = -\text{min}(0, P_S + P_D - P_L + P_{slack} + P_E) P_I(t, P_D) = \text{max}(0, P_S + P_D - P_L + P_{slack} + P_E) \tag{4}$$

$$SOC(t) \in [0.2, 1] \tag{5}$$

$$P_D(t) \in \{0\} \cup [P_{min}, P_{max}] \tag{6}$$

$$\begin{cases} P_I(P_D(t), t) \in [0, 13.2] & \text{if} \quad SOC(t) < 0.9 \\ P_I(P_D(t), t) \leq 1320 * (SOC(t) - 1)^2 & \text{otherwise} \end{cases} \tag{7}$$

$$P_O(t) \in [0, 40] \tag{8}$$

The Optimal Control problem, where $x(t) \equiv SOC(t)$ and $u(t) \equiv P_D(t)$, can thus be stated as:

$$\begin{cases} \text{Minimize} & J[u] = \int_0^T (Ku(t)^{0.9} + c_E^* P_E)dt \qquad \text{from (3)} \\ \\ \text{s.t.} & \dot{x(t)} = f(u(t), t) \qquad \text{from (1)} \\ & u(t) \in U_{x(t)} \qquad \text{from (6)} \\ & x(t) \in C \qquad \text{from (5)} \end{cases} \tag{9}$$

1.3 Problem Formulation

Based on the following assumptions, a few constraints have been relaxed:

- No slack: assuming all the power is being used up and any lack of power is being fulfilled by external power.
- P_D can take any value between 0 and P_D^{max}
- Battery cannot have input and output at the same time. This has been generalized as P_B; if P_B is negative, the battery is discharging and charging if positive. The efficiency being included accordingly in either cases.
- Upper bound on input battery power is taken to be independent of state of charge.
- Switching cost on diesel power is ignored.
- No constraint was applied on the final state of charge of battery.

Therefore the governing equations become:

$$\dot{SOC}(t) = \frac{P_B(t)}{Q_B}(0.8^{\frac{abs(P_B(t))}{P_B(t)}}) \tag{10}$$

$$P_D + P_S + P_E - P_L - P_B = 0 \tag{11}$$

The cost function is given by:

$$\int_0^T (KP_D(t)^{0.9} + c_E P_E(t))dt \tag{12}$$

Therefore, the new Optimal Control Problem can be stated as:

$$\begin{cases} \text{Minimize} & J[u] = \int_0^T (KP_D(t)^{0.9} + c_E P_E(t))dt \qquad \text{from (12)} \\ \\ \text{s.t.} & \dot{x(t)} = f(u(t), t) \qquad \text{from (10)} \\ & u(t) \in U_{x(t)} \qquad \text{from (6)} \\ & x(t) \in C \qquad \text{from (5)} \\ & P_B(t) \in [-40000, 13200] \\ & P_E(t) \geq 0 \end{cases} \tag{13}$$

2 Numerical Methods

Simulations were performed with the given data sets. Based on the data set, the predictability of production of power from different sources, including the external power sources, using the deterministic or stochastic modeling was determined. The following three methods were used to approach the problem and obtaining the optimal solutions:

- Forward Euler Method
- Dynamic Programming Method
- Collocation Method

2.1 Direct Simultaneous Method with Forward Euler Scheme

2.1.1 Methodology

The solution of the differential equation at each time step t_k is obtained sequentially using current and previous information about the solution. The Forward Euler Method is a time-marching multiple-step method, where the solution at time t_{k+1} is obtained from a defined set of previous values. It is one of the most common single-step methods and has a general form:

$$x_{k+1} = x_k + h_k f_k \tag{14}$$

where $f_k = f(x(t_k), u(t_k), t_k)$ This is an explicit method (because the value $x(t_{k+1})$ does not appear on the right-hand side of (14). The solution is less stable than that obtained from implicit methods like Backward Euler or Crank-Nicolson, but it requires lesser computation at each step.

To implement this method, we converted the problem into Meyer form with derivative of cost as one of the state. The other state was state of charge of battery and control were diesel power and external power. Battery power was assigned as a intermediate variable.

2.1.2 Results

The optimal cost that was obtained from the Forward Euler scheme is 1.4087e+06, the states and controls are as follows:

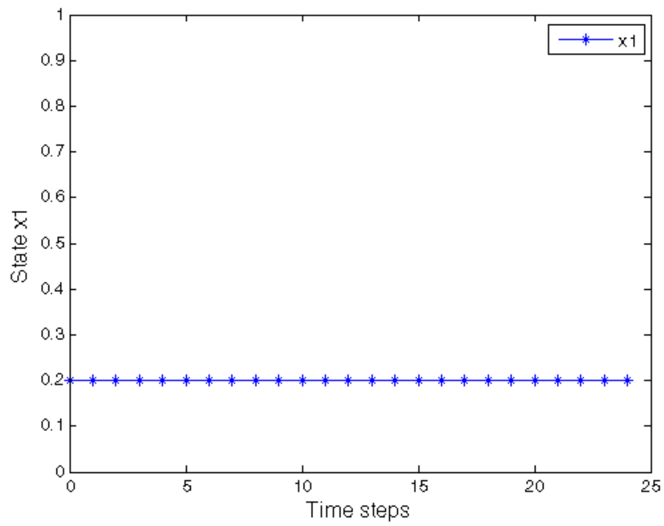

Figure 1: State of charge vs Time

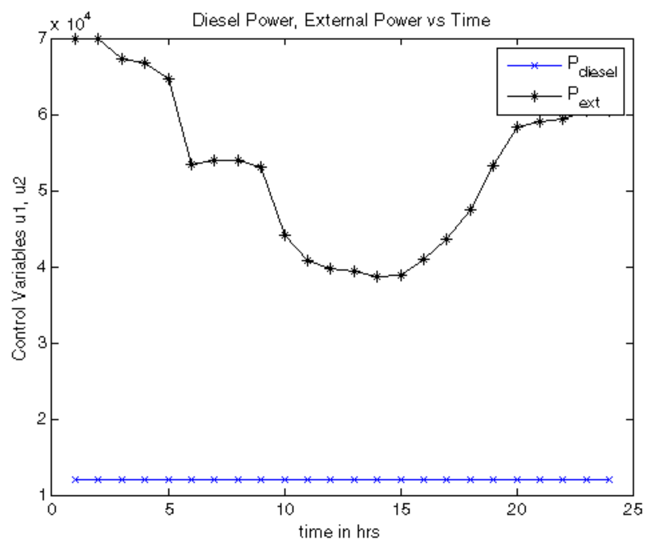

Figure 2: Control Variable (P_D) vs Time

2.2 Dynamic Programming Method

In order to solve the DPP, we propose a semi-Lagrangian scheme. In addition, the Pontryagin Maximum Principle (PMP) can be used to give additional information on the optimal solution, which allows reducing the computational effort of the method significantly.

2.2.1 Brief Presentation of the Theory

Let $V(t, x_0)$ denote the value of problem (OCP) with initial time t and initial condition x_0. In R. Bellman's words, "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." In mathematical terms, V satisfies for $h \in (0, T-t)$:

$$V(t, x_0) = \inf\Big\{ \int_t^{t+h} l(u_t) ds + V(t+h, x(t+h)) \Big\} \tag{15}$$

the infimum being taken over the set of admissible controls. In our framework, we will use an extended version of the DPP approach that handles the switchings.

2.2.2 Semi-Lagrangian Scheme

The Semi-Lagrangian scheme consists of solving a discretization of (15) over the space, backward in time. We have chosen this scheme to solve the problem because it has good stability properties, it allows large time steps and it is easy to implement. Let us motivate the scheme by first discretizing (15) in time. Given a time step h and N such that $Nh = T$, let us set $t_k = kh(k = 0, 1, 2, ..., N)$. Denoting by V_k the "approximated" value function at t_k we have:

$$V^k(x) = \min_{u \in U_x} \big\{ hl(u) + V^{k+1}(x + hf(u, t_k) \big\} \tag{16}$$

The Semi-Lagrangian scheme is obtained from (16), by discretizing in space the state variable x and introducing interpolation operators in order to approximate $V^{k+1}(x + hf(u, t_k))$ in terms of its values in the space grid. The scheme is solved backward in time and, under standard conditions, it is shown that it converges to the solution V of (15).

2.2.3 The PMP Trick

The problem has an interesting property that greatly reduces the numerical computations. If u^* is the optimal control, x^* be the optimal state and p^* the Lagrange multiplier associated with the dynamics constraint $\dot{x}(t) = f(t, u(t))$. Defining the Hamiltonian $H(u, p, t) \equiv pf(u, t) + l(u)$, the PMP says that,

$$H(u^*, p^*, t^*) \geq H(u, p^*, t^*) \quad \forall \quad t \in [0, T] \quad and \quad u \in U_{x^*} \tag{17}$$

Since $u \to H(u, p^*, t^*)$ is strictly concave piecewise, it can attain its minimum at only one of the extreme points of the pieces. Also taking the constraints into account, we have at most five possible optimal controls. Moreover, the values of those controls can be computed explicitly, since they do not depend on p. Therefore, when doing the minimization in (16), we can test only those controls instead of discretizing the control space, gaining both in speed and precision.
We test the five cases:

1. $P_D = P_{min} = 0$ (minimum power),
2. $P_D = P_{max} = 12000$ (maximum power),
3. P_D such that $SOC = 0$ (battery unused),
4. P_D such that $P_B = 13.2 = P_B^{max}$, (maximal charge),
5. P_D such that $P_B = -40$ (maximal discharge)

It should be noted that the specific structure of the problem permits such computational simplification. More precisely, we use the fact that all the candidate values for the optimal control do not depend on the adjoint state p and therefore can be evaluated and tested when computing the value function. In the general case, the control that minimizes the Hamiltonian is expressed from both the state and adjoint state, the latter being unavailable in the DPP approach (the adjoint actually corresponds to the gradient, with respect to the state variable, of the value function V^k).

While implementing this method, choices of control were limited to a discrete set of controls, obtained from PMP. Controls were taken as diesel power and battery power, and their values limited to the cases mentioned above. Minimum cost was calculated while back-stepping from the terminal stage to the initial stage and finding the optimal path.

2.2.4 Results

The results that were obtained from applying the DP method to the problem are as follows:

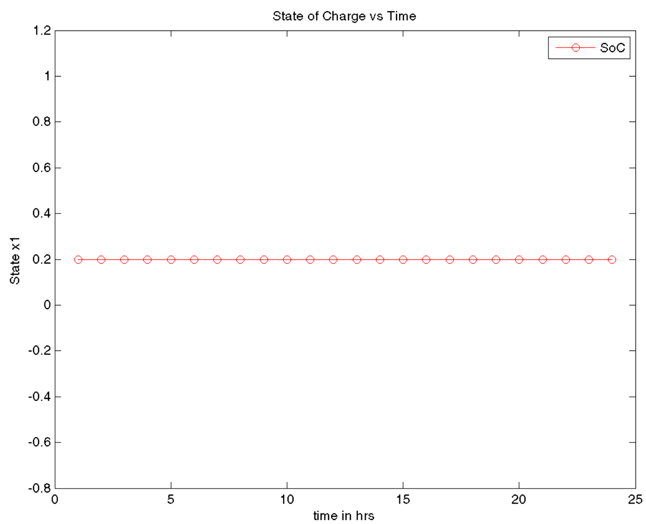

Figure 3: State of Charge vs Time

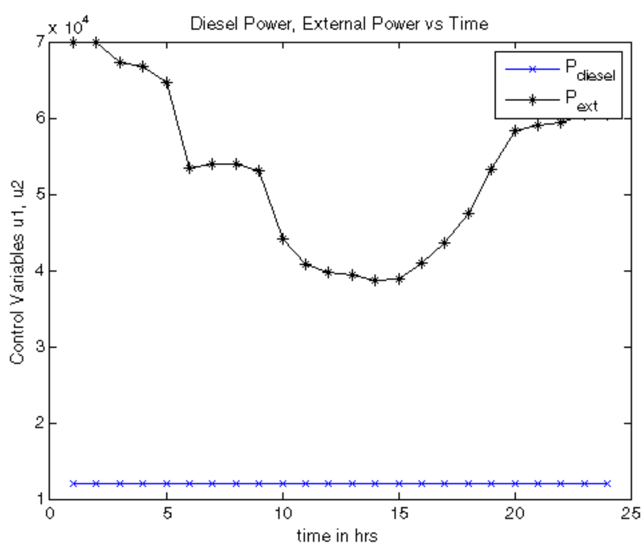

Figure 4: Diesel Power (P_d) and External Power (P_E) vs Time

2.3 Collocation Method

2.3.1 Methodology

PROPT is a software package run on Matlab, intended to solve dynamic optimization problems. Such problems are usually described by:

- A state-space model of a system. This can be either a set of ordinary differential equations (ODE) or differential algebraic equations (DAE).
- Initial and/or final conditions (sometimes also conditions at other points).
- A cost functional, i.e. a scalar value that depends on the state trajectories and the control function.
- Additional equations and variables that, for example, relate the initial and final conditions to each other.

PROPT uses pseudo-spectral collocation methods for solving optimal control problems. The solution takes the form of a polynomial, and this polynomial satisfies the DAE and the path constraints at the collocation points (Note that both the DAE and the path constraints can be violated between collocation points). The default choice is to use Gauss points as collocation points, although the user can specify any set of points to use.

The problem was converted into Meyer form. State of charge and derivative of cost were taken as the state variables and diesel power and external power were taken as control. Both the states and controls were discretized using collocation method. The problem was modeled, discretized and optimized using *Tomlab PROPT software*.

2.3.2 Results

The optimal cost was found to be 1.3984e+06, and the controls and states are as follows:

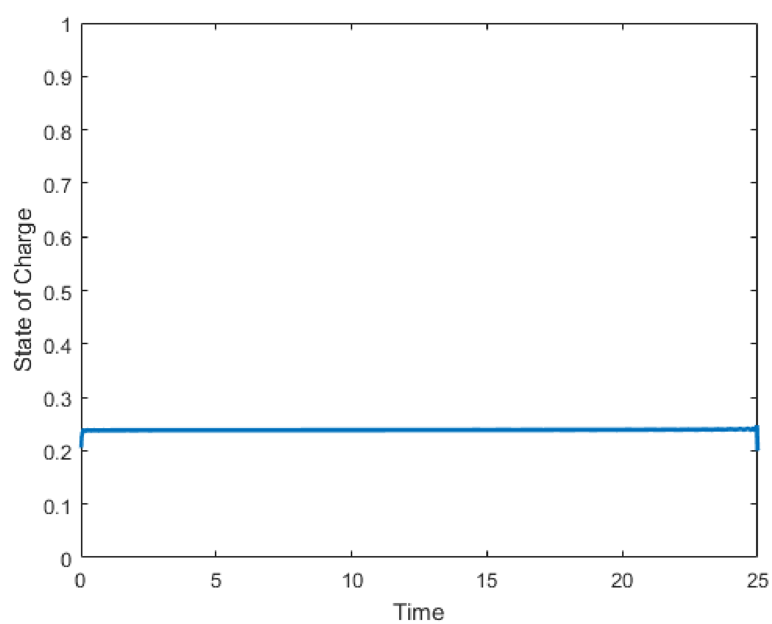

Figure 5: State of Charge vs Time

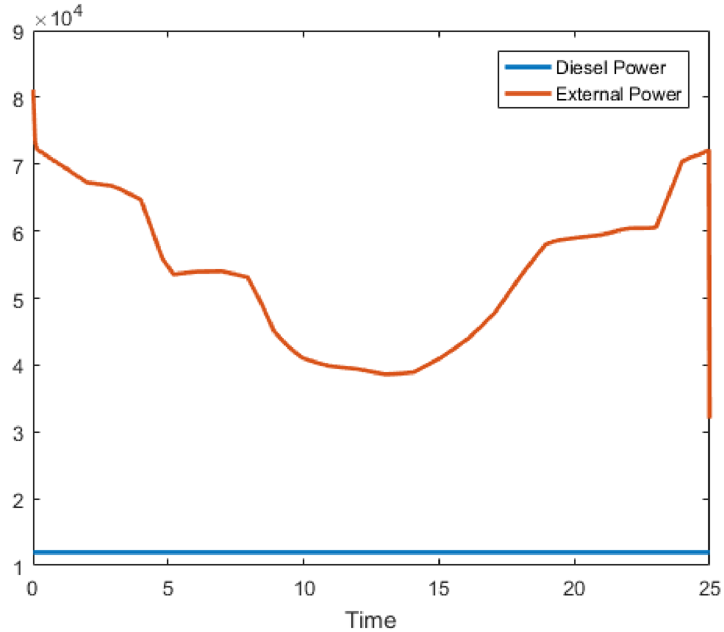

Figure 6: Diesel Power (P_D) and External Power (P_E) vs Time

3 Conclusion

- Since there is no cost associated with solar power, it is always being used. The remaining power is first supplied by diesel power and then by external power, as the unit cost for diesel power is less than that of external power.
- All the three methods give approximately same minimum cost for one day's operation.
- The computation time for all the methods were high though the Collocation method gave the fastest results and the Euler method gave the slowest results. The optimization was done for only a day's data, as the computation for the month's data would have taken a very long time.

4 References

1. Benjamin Heymann, J. Frederic Bonnans, Pierre Martinon, Francisco Silva, Fernando Lanas and Guillermo Jimenez-Estevez
   Continuous Optimal Control Approaches to Microgrid Energy Management
   URL https://link.springer.com/article/10.1007/s12667-016-0228-2