

GrabBin

Retrofittable sweeping module for public bins

A Project Report

submitted by

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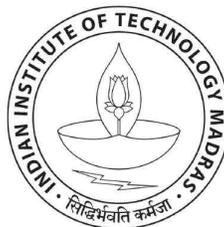
in partial fulfilment of the requirements

for the course

ED3110: PRODUCT DESIGN LAB-II

Course Guide

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Abstract

A device for periodically sweeping the area in front of a municipal garbage bin was designed and fabricated in this project. The device is entirely mechanical in operation and is intended as a retrofittable addition, with the purpose of facilitating access to the bin. A module which converts linear motion of falling dead weight to power the actuation mechanism are designed and fabricated. Also a method for timing the actuation by using the principle of hour glass has been proposed.

Keywords - Garbage, mechanical actuation, municipal solid waste management, urban hygiene, four bar mechanism, coupler curve, mechanical timing

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1 Introduction

In cities of ever expanding population like the Indian metropolitan cities, the amount of garbage generated is increasing in every neighborhood and locality. This causes a major problem in areas where garbage collection happens through large community dustbins, namely the scattering of garbage around these dustbins due to overflowing bins, human negligence and animal interference. This is an unsightly and unhygienic affair which affects the aesthetic appearance of the urban environment, and causes community problems like breeding of pathogens, disease vectors like mosquitoes, flies and rats, and gathering of stray animals (?).

Due to the sheer number of dustbins present in a city, a large amount of manual labor is required just to clean the areas around each dustbin. Moreover, this cleaning happens only during garbage collection. The objective is to address this issue with a fully automatic mechanical system that periodically cleans the area around municipal dustbins and yet be robust and accessible.

1.1 Problem Statement

To design an accessory product retrofittable to a municipal garbage bin, which ensures cleanliness in the area surrounding the bin and is both economically and technically suitable for large-scale implementation in cities.



Figure 1: Overflowing Dustbin
Source: Google images

1.2 Motivation

The accumulation of garbage in public spaces is self-perpetuating. Consistently dirty streets or corners lead to public apathy, which in turn leads to more littering. Generally it is observed that sight of a littered part of a street increases the probability of an observer contributing to the litter suggesting that any means of reducing the amount of litter in a public place would have a positive effect on littering trends, and an active counseling accompanying it to the members of the society would further reduce public littering.

An additional factor concerning urban solid waste collection in India is that litter is found not only at street corners, but also surrounding municipal garbage bins themselves. This has been observed by the authors even in areas where garbage collection is regular, and even around bins which filled to half or less of their capacity. The reasons might be twofold: initially, littering occurs due to the laziness or apathy of some. As a result, others are affected not only by the subconscious relaxation described above, but also by an added physical difficulty in accessing the garbage bin. While this difficulty may be surmountable with some effort, the fact that this effort is required at all, to dispose of the garbage in the bin, leads to more users disposing of it outside, adding to the accumulated garbage surrounding the bin.



Figure 2: A dustbin in Taramani, Chennai

1.3 Justification

- Community bins are a common means of waste disposal in Indian cities. For example, 17.5% of commercial areas in Bangalore have community bins according to an audit conducted by the Indian Institute of Science.[1]
- The above report also lists 20 problems associated with garbage being exposed, such as attracting stray dogs, rats and cows, providing a breeding ground for flies, mosquitoes and other insects, which cause spreading of diseases.
- As garbage is just flung into dustbins, there is usually lot of empty pockets even in full dustbins, so there is scope for compacting garbage and creating more space. This is done in dustbins in some foreign countries.

2 Design Methodology

2.1 Prior Art

A patent search was done to explore the various existing approaches to the problem. The links to some relevant patents are given below:

1. Bulk material removal: Addresses the problem of untimely garbage pickups. <https://patents.google.com/patent/CN2464681Y/en?q=dustbin&q=scatter&scholar>
2. Addressing the problem of falling dustbins and scattered content: <https://patents.google.com/patent/GB2211397A/en?q=dustbin&q=scatter&scholar>
3. Design of a dry trash can to prevent odour and to make it comfortable to access: <https://patents.google.com/patent/CN203255589U/en?q=dustbin&q=scatter&scholar>
4. Multifunctional dustbin: Addresses multiple problems by means of “smartness” <https://patents.google.com/patent/CN201494853U/en?q=dustbin&q=scatter&scholar>
5. Intelligent bin: <https://patents.google.com/patent/CN203512491U/en?q=dustbin&q=scatter&scholar>
6. Automatic sealing for garbage bins: Achieved using gravity <https://patents.google.com/patent/CN2405905Y/en?q=dustbin&q=scatter&scholar>
7. Electromagnetic Sealing Mechanism: <https://patents.google.com/patent/CN201385912Y/en?q=dustbin&q=scatter&scholar>
8. Trash Bin: <https://patents.google.com/patent/CN203006213U/en?q=dustbin&q=scatter&scholar>
9. Sanitary and convenient bins: <https://patents.google.com/patent/CN2858576Y/en?q=dustbin&q=scatter&scholar>
10. Dustbin with a filling device: <https://patents.google.com/patent/CN2584564Y/en?q=dustbin&q=scatter&scholar>

2.2 Scope

This is an active area of research with multiple perspectives to solve the problem as seen in the literature survey. The current situation with regard to the current designs of garbage bins in the country is that while solutions have been proposed to improve waste collection and disposal, they have not been implemented on a reasonable scale in most cities owing to vulnerable components. While existing designs for Indian municipal bins use sensing to improve waste detection or software provide notifications for timely removal, they require drastic changes in implementation to be effective and would be much apt in a smart city setting which needs a lot of reforms and infrastructure to maintain them.

The intention here was to address not the problem of timely garbage collection or organizational issues in the municipality, but the disposal of garbage in the bins itself. The design process was initially kept open to exploring mechanical, mechatronic, socio-psychological and other approaches, with the understanding that preferably, any moving parts would require low power and would be actuated by the weight of the garbage.

It was identified that hygiene in the area surrounding the bin could be ensured by either prevention of littering around the bin, or periodic removal of litter by the device. The scope of the project was defined to include consideration of both options to create a design that would be (a) possible to implement within the cost constraints of municipal waste departments (b) effective regardless of civic will and (c) retrofittable.

2.3 Design Challenges

The problem, by its nature, imposes a lot of constraints in designing a solution. Apart from the usual cost constraint, the main challenge here was to ensure the usage of cheap and simple components. Given the working conditions of the product (in streets and roadsides), usage of electronic or any complex means for energy generation, storage and actuation would also mean running the risk of having the components stolen. The electronic components (for e.g., batteries, solar panels, motors etc.) will also require protection from moisture, heat etc. for proper functioning, apart from the fact that they are costly.

- The energy module has to source and store a significant amount of energy. This has to be done from the very few energy sources available. This process has to use minimal human involvement, or preferably, no involvement.
- The cleaning operation has to be triggered at an appropriate moment, and the trigger has to function without any standard sensor (since most of them are electronic).
- Any major change to the existing waste management process, and any structural modification to the existing bins will greatly reduce the chances of the solution being implemented.

It has to be noted that very few solutions from the prior art involve completely non-electronic means for functioning. Most of the existing solutions involve motorized actuation. Some solutions use sensors along with IoT (Internet of Things) technology. It is the opinion of the authors that these solutions will find limited success in most Indian streets.

Relatively cheap, passive and mechanical retrofittable solutions were generated in the concept generation phase, as they can overcome most of the above challenges. The concepts are explained in a later section.

2.4 Mission Statement

Product Description	A product which ensures that the area surrounding the dustbin is free of litter, and makes the place more approachable for disposing of garbage.
Key business or humanitarian goals	Design a working prototype by the end of the lab period within the given budget constraints.
Primary market	Municipal Corporation
Secondary Market	Any community based residential area
Assumptions	Should not hinder the present capabilities and should be simple to augment to the present design. Shouldn't interfere during the waste collection from the bin.
Avenues for creative design	Simple automated system using obvious energy sources. Retrofittable to existing bins. The corresponding mechanism or means of achieving the action.
Scope Limitations	Shouldn't interfere with animals etc., the region of access is limited, very rough and undulated terrains might not be compatible.

2.5 Design Specifications

- The module(s) cannot rely on the chassis of the bins for reference or support, rather it must be designed to refer to sturdier components such as the steel pins for the collector truck.
- The collectors have to be made aware of placing the bin in an appropriate orientation so as to efficiently collect the garbage from the module.
- The approximate weight of the garbage lying around the bins that has to be collected in one cleaning cycle is about 15 kgs.
- The radius of cleaning around the bin should be approximately 500 mm.
- The amount of force required to compact the garbage is too large to generate from a passive energy source.
- The terrain around the bins is highly varying and the cleaning mechanism will have to incorporate this into the design.
- Energy required for collecting the lying garbage:

$$\text{Energy } E = m * g * h = 15 * 9.8 * 0.52 \approx 80J$$

Keeping a safety factor of 2.5, total Energy required for cleaning

$$E_{total} = 200J$$

- Area around the bin to be cleaned:

$$A = w * r + 0.5 * \pi * r^2 = 1.04 * 0.5 + \pi * 0.5 * 0.5 * 0.5 \approx 0.9127m^2$$

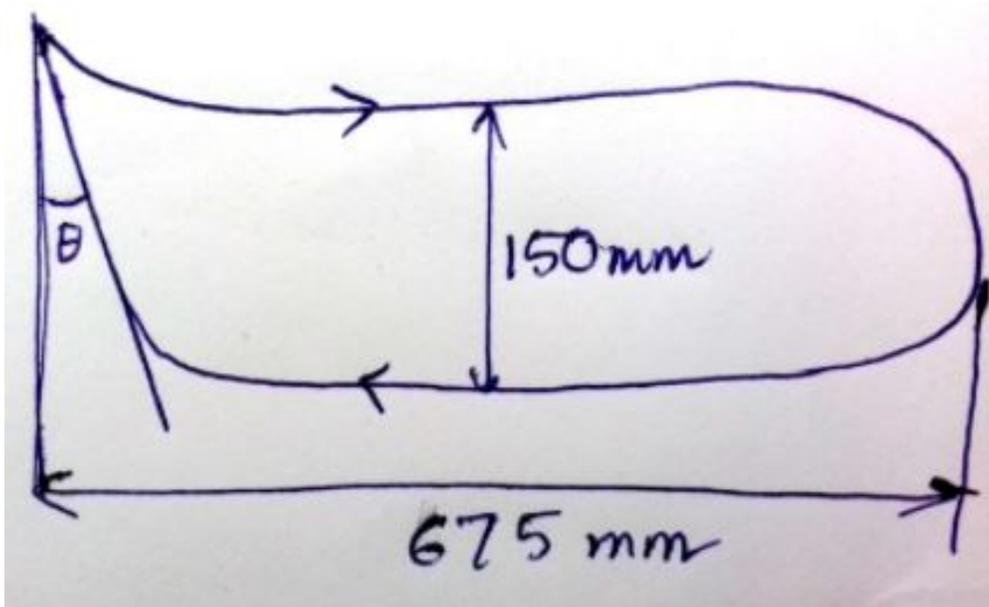


Figure 3: Design constraints for the sweeping curve

- Mass of dead-weight

$$M_w = \frac{E_{total}}{g * h} \approx 20kgs$$

- The dimensions of the dustbin are as follows:

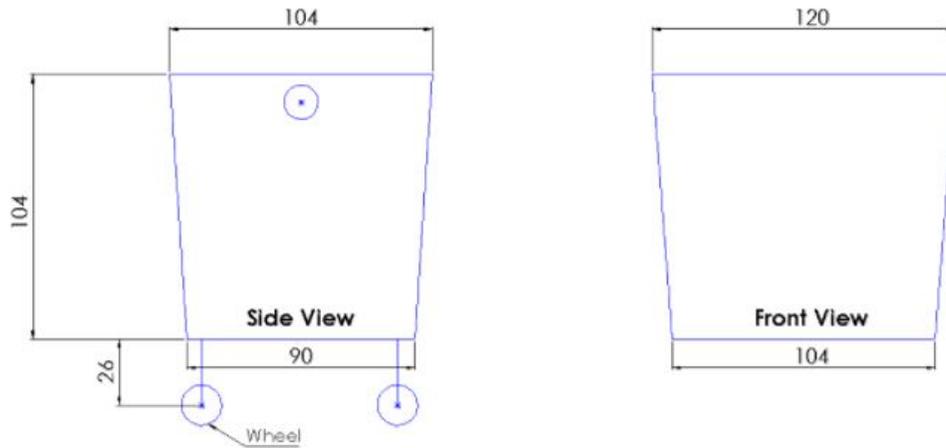


Figure 4: Dustbin dimensions

2.6 Target Customer

The target customers for the proposed product are the various municipal corporations around the country that can make use of the said product to either upgrade or replace their existing community bins. As indicated in the Central Pollution Control Board annual report cited above, there are a lot of states that have municipalities which lack adequate garbage collection infrastructure.



Figure 5: Municipality workers collecting the garbage from the bin

2.7 Data Acquisition

To verify the claims in the initial problem statement and to estimate some parameters in the processes involved, data was collected from the following sources:

- For most efficient garbage collection, the garbage lying around the dustbins must also be collected.[2]
- The density of waste (mass per unit volume, kg/m^3) determines the storage and transportation volume requirements. MSW density in India is typically around $450\text{--}500 \text{ kg}/\text{m}^3$. [3] Source-
- A survey was sent out where participants were asked to rank the following four problems faced at a community dustbin in the order of their severity (rank-ordering for getting pseudo-quantitative data)
 - Garbage lying around the dustbin
 - Animals that gather around due to the garbage
 - The dustbins overflowing
 - The odor and insects that collect there.

The survey results follow.

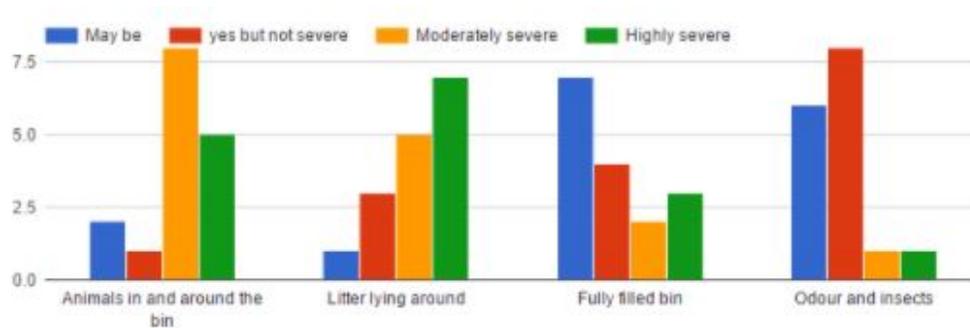


Figure 6: Overview of survey results

3 Concept Generation

Based on the required functions in the product, it was subdivided into the following modules:

- Energy generation module
- Timing module
- Cleaning Mechanism

3.1 Energy Module

3.1.1 Battery Pack

Using a battery pack is not a feasible solution due to high possibility of theft, limited life, and non availability of a source of electrical energy for recharging of batteries.

3.1.2 Solar Panel

This is not a feasible solution due to risk of theft, blockage of the solar panel surface, lack of sunlight and limited sunlight.

3.1.3 Gravitational Energy

A freely falling weight can act as a source of energy for collection of garbage. Initially the mechanism is at the top of the dustbin. A timer holds the weight in place and releases it only after a specific time delay, after which the weight falls downward.

The weight will once again be set at the top when the garbage truck inverts the dustbin to collect garbage. The mass of the suspended weight attached should be small as compared to the mass of the dustbin as it should not obstruct the functioning of the garbage truck.

3.1.4 Garbage Powered Turbine

This concept employs the weight movement mechanism from the above idea and uses it to solve the additional problem of compaction.

For lifting the weight, energy of the garbage coming in from the top of the bin (which is more in frequency and volume) is harnessed by letting it fall through the face of blades fixed in slots of the thin shaft shown. The shaft is connected to the actual mechanism (red triangle) which is housed under the ramp for protection from rain etc. which includes components like a gearbox, freewheeling device etc. for lifting the weight directly under it and holding.

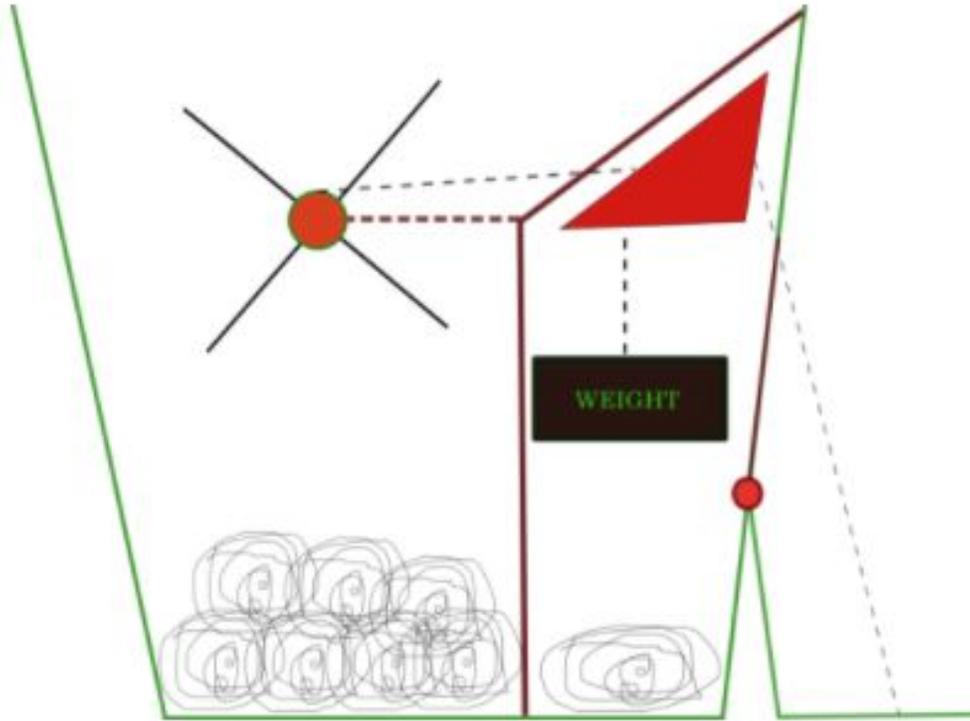


Figure 7: Turbine power concept

Once the weight crosses a threshold height, a switch in the mechanism releases the weight and engages the collector arm which contains the garbage strewn around the bin (low volume). The falling weight makes the arm rotate towards the bin and throw the said garbage into the smaller compartment under the weight. Eventually the weight falls over it and stays till it is pulled up by the mechanism later, compacting it in the process.

Another advantage of this concept is that the separating wall enables the device to lift the garbage only for a smaller height (lesser energy required) rather than doing the same all the way to the top of the bin.

3.2 Timing Module

The timing module can be either an electronic timer coupled with an actuator, a mechanical clockwork system or an hourglass.

3.2.1 Mechanical Clockwork

A mechanical clockwork is very intricate and delicate and can malfunction due to even small amounts of moisture and dirt. Thus it is not suitable to use.

3.2.2 Electronic Timer

An electronic timer with an actuator is simple, cheap and reliable but it requires a source of electric energy and an electronic trigger, which is not available.

3.2.3 Hourglass

A sand timer is convenient as the amount of sand required to achieve a time delay of 6-7 hours is very close to 20 kgs, which can serve as the weight for actuation. However, it is susceptible to moisture and requires a fairly

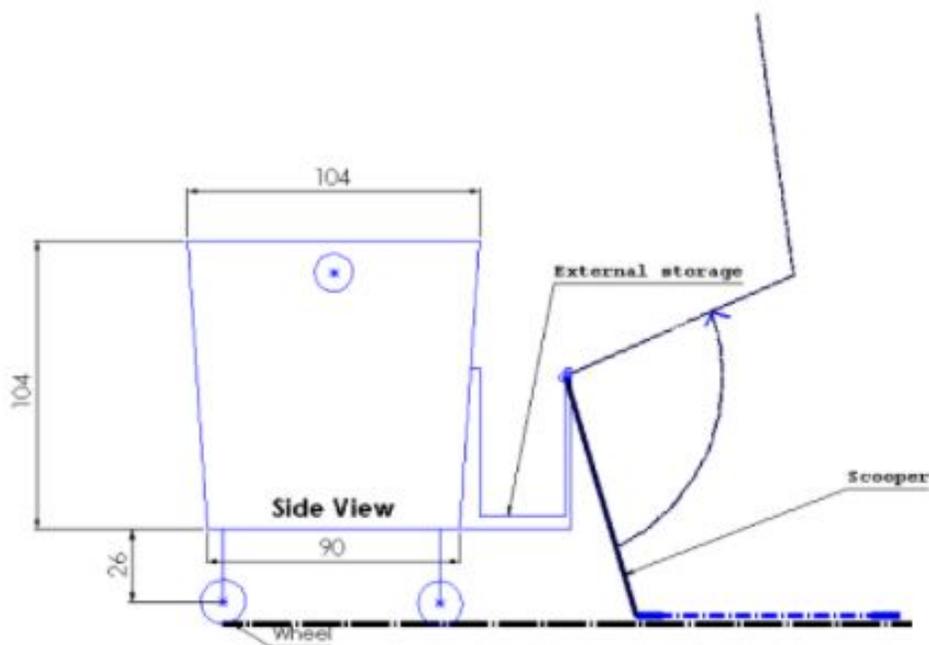
complicated mechanism to hold/drop the weight depending on the level of the sand.

3.3 Cleaning Module

3.3.1 Lifting Platform

In this mechanism, an L-shaped sheet lies idle in front of the mechanism. One side lies flat on the ground and the other is vertical, supported by the bin wall. It is hinged at its top with the bin. The horizontal sheet lies under the accumulating garbage outside the bin. (Note : To reduce the weight of this component, it can be made from a sufficiently strong flexible mesh, with some minimal solid reinforcements)

At an appropriate time, the mechanism rotates the whole part, along with the garbage (using ropes attached at fixed points, pulled at the other end by the falling weight before passing through a pulley). This action lifts the garbage outside and lets it roll/fall into the reservoir from the top.



Schematic depicting the scooping action of the L-shaped sheet

Figure 8: Lifting platform concept

Advantages are simplicity of operation, which also results in an almost trouble free working. It suffers mainly from the following disadvantages:

- The weight has to be lifted through a height, against gravity, which requires significant energy. This may not be necessary.
- It occupies some space outside the dustbin. (However this is not a serious issue since it lies flat on the ground, under the garbage, possibly indistinguishable from usual conditions).
- It may encourage some people to dispose their garbage on it, which is easier, rather than spending effort to throw into the main bin. This problem is partially solved by having the mesh (see note above), which makes it almost transparent, and hence partially hidden from the eyes of a casual observer.

3.3.2 Guided Scraper

The path-following cleaning mechanism involves a sweeping arm that follows a specified path to move over the garbage when moving outwards and then lowering to the ground level before moving back towards the dustbin, thereby scraping the garbage towards the bin.

The scraper is made of a compliant material similar to that found in plastic broomsticks so that it can move over very rigid/heavy objects in its pathway while it is in motion. The path of the scraper is mechanically pre-determined and it is a passive mechanism. The path is so designed to move the scraper above the garbage lying in front of the module when it is moving out from the module. As it is reaching it's maximum distance away from the module, it lowers to the ground level and begins to move towards the bin. As the scraper completes it's backwards motion, it also begins to move up, pushing the garbage up a ramp that helps collect the garbage into the module. Finally, when the garbage has been collected, the scraper arm moves back to its starting position.

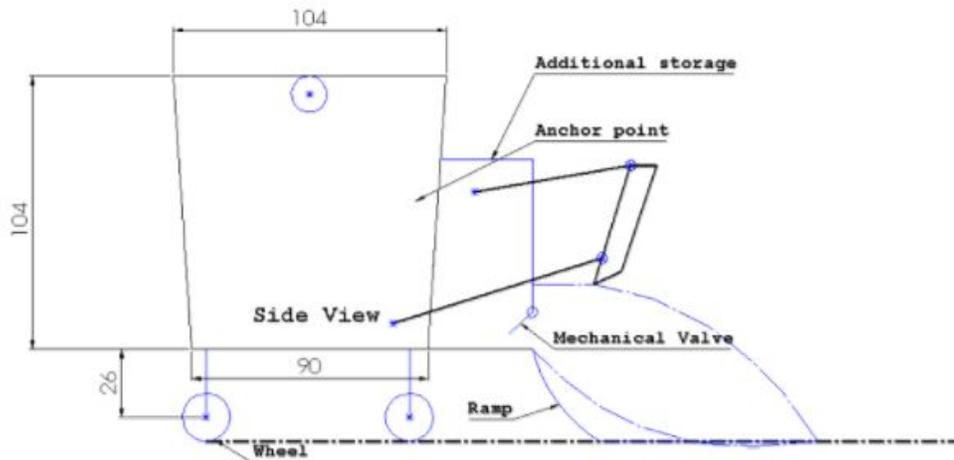


Figure 9: Guided scraping concept

The advantages of this mechanism are:

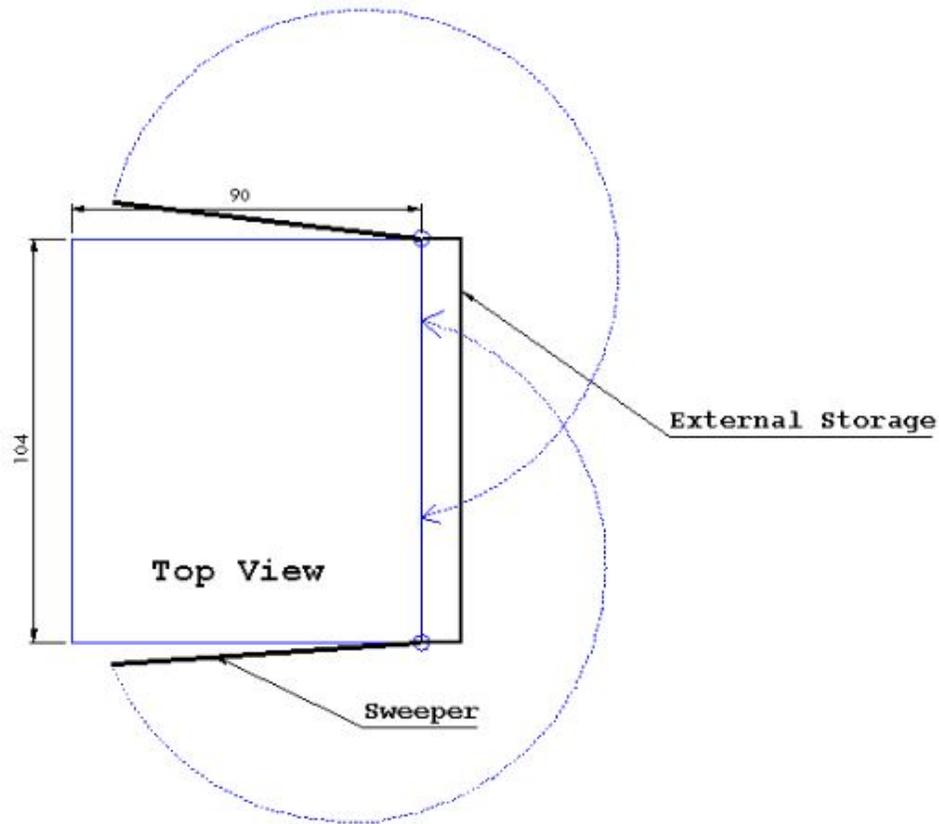
- It is compact in its motion, having a very restricted operating region
- Its home position is inside the module, hence it does not hamper the area around the bin unless necessary
- It can deal with obstacles in its operating region much more efficiently than the other proposed mechanisms

The disadvantages of this mechanism are:

- It has very high mechanical complexity as compared to the other proposed mechanisms
- If the mechanism fails while in operation, it shall remain in an extended position till the garbage collectors come to reset it.

3.3.3 Simple Sweeper

The sweeping mechanism consists of two simple sweeping arms that move out from either side of the retrofittable module. These sweeps have brushes along their length to bypass any rigid objects in its path and yet can sweep all the garbage along with it. The sweep is actuated by the energy module. It only differs in the area covered by the path but the actuation and garbage collection mechanisms are very similar to that of the path generation concept.



The path of the sweeper as shown from the top view

Figure 10: Top view of sweeping action

The advantages of this mechanism are:

- A larger area can be covered as the workspace consists of circles whose radius can be altered.
- It can collect garbage within a wider span angle than the other two proposed mechanisms.
- Simpler actuation means as a simple rotary joint would be enough to drive the arms.

The disadvantages of this mechanism are:

- If any larger garbage chunks or hard items are in its way, it might get stuck there until rectified. This would obstruct a user from freely accessing the dustbin and hinders users from throwing the garbage inside the bin.
- Then it would lead to counterintuitive effects and increase the garbage around the bin more than usual.
- As it spans for a larger angle it has more probability to get stuck on its path which is near to the ground surface unlike the guided scraper which comes down from a certain height to pick the garbage.

4 Concept Selection

4.1 Energy Module

The concept of using garbage weight to power a turbine was deemed to be not feasible because of variable weight of garbage that may not rotate the turbine but instead may overflow from the turbine itself.

The concept of using falling weights as an energy source is simple to implement and has an advantage that it gets automatically reset to the top when the garbage collection truck inverts the dustbin to collect the garbage. However, it has a disadvantage that it relies on linear sliders for movement, which may get damaged by impact forces and torque. This can be minimized by using linear sliders sliding on cylindrical rods, which are structurally sound, which led us to choose this concept over others.

4.2 Timing Module

The timing module was selected to be the hourglass concept as it satisfied the requirement of being a purely mechanical timer and also acts as the dead-weight for the energy module.

4.3 Cleaning Module

From the 3 concepts which were specified above, a concept screening was done to finalize on one method of collection of the garbage from the vicinity.(rank ordering, 1 being best and 3 being poor)

Factors	Concepts		
	Lifting Platform	Guided Scraper	Sweep Type
Robustness	1	2	3
Simplicity	1	3	2
Compactness	3	1	2
Cost	1	1	1
Accessibility	3	2	1

From the scoring pattern it was decided that the guided scraper type mechanism will be pursued owing to its robustness and compactness.

5 Prototype Description

The components used in the design include two sliders and two four bar mechanisms attached on either side. The overall function of the prototype is as follows: A weight slides vertically on two linear bearings which actuate a pair of mild steel 4-bar linkages through a chain-sprocket mechanism. The chain, which is connected to two sprockets at the top and bottom of the slider, is also rigidly clamped to the weight. This causes the downward motion of the weight to be converted to rotational motion of the sprockets, which is then transmitted to the four bar through polypropylene shafts.



Figure 11: Photograph taken of the prototype



18
Figure 12: Another photograph taken of the prototype

6 Analysis

6.1 Dynamics of the four bar mechanism

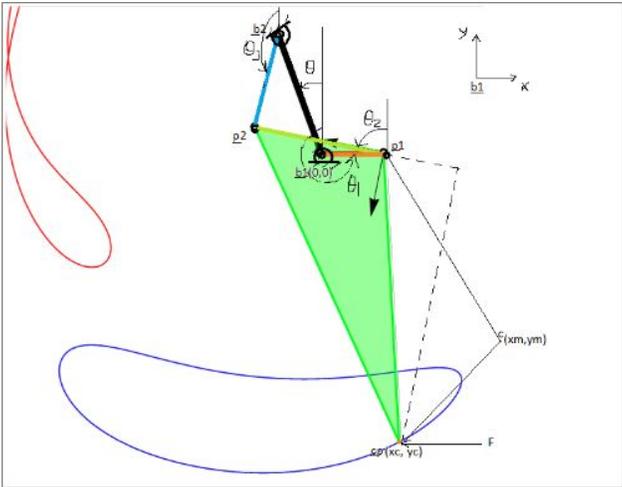


Figure 13: Illustration of 4 bar mechanism

The following analysis was done in Wolfram Mathematica (code in appendix-A). The variations of the coupler angle and follower angle with the input angle were plotted and it was noted that the coupler link follows a smooth profile with respect to the input angle, over the sweeping cycle.

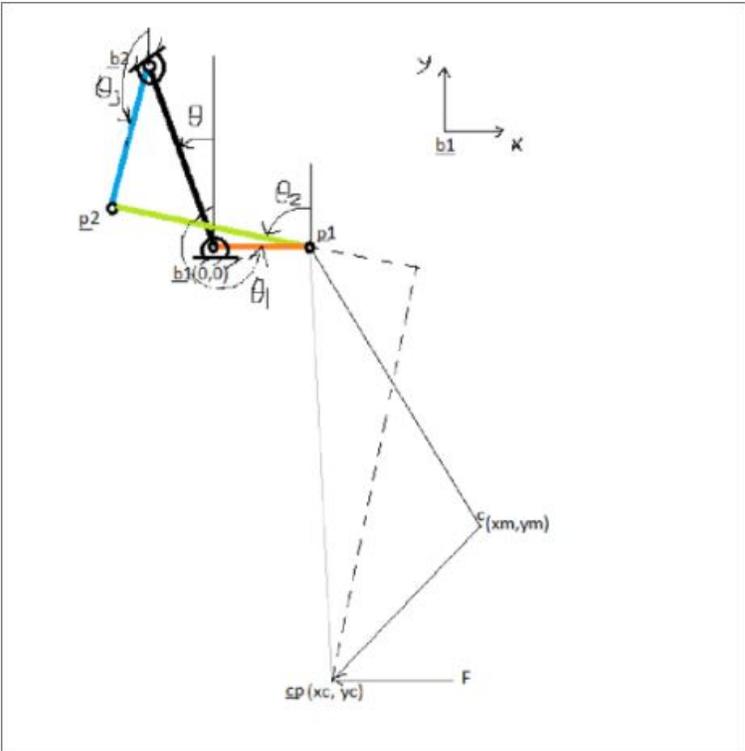


Figure 14: Illustration of 4 bar mechanism

Assuming that a force of 75 N is required to sweep the garbage, the torque required at the input joint was calculated as a function of input angle, when (x_m, y_m) lie along the coupler link.

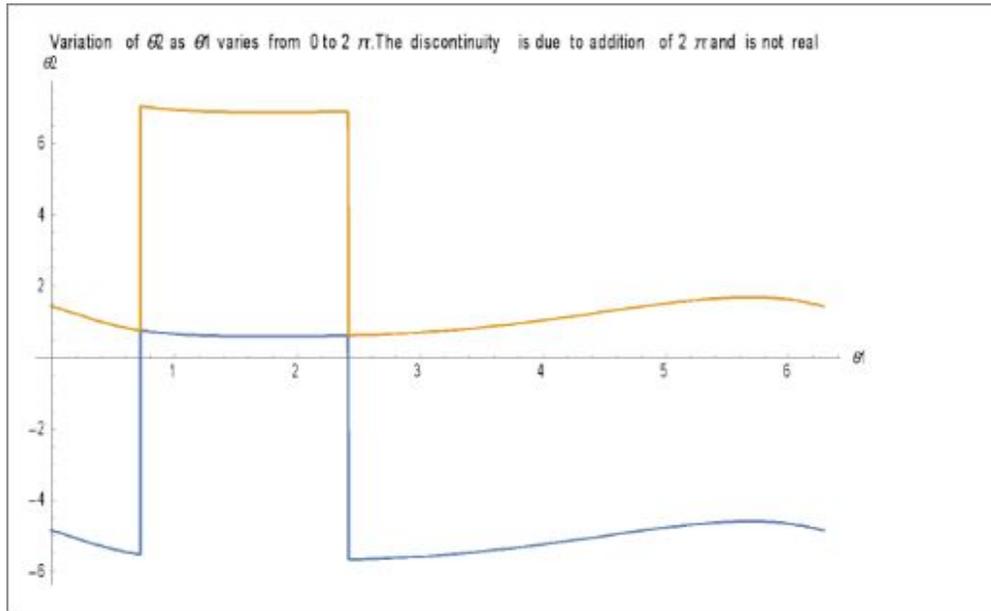


Figure 15: Smooth variation of coupler link angle with input angle over the sweeping cycle

An attempt was made to optimise the shape of the coupler extension to minimise this torque, by creating contour plots of the maximum torque over varying (x_m, y_m) ; however, this had a negligible effect on the maximum torque and hence the location of (x_m, y_m) along the coupler link with the extension at a right angle was chosen for ease and accuracy of fabrication.

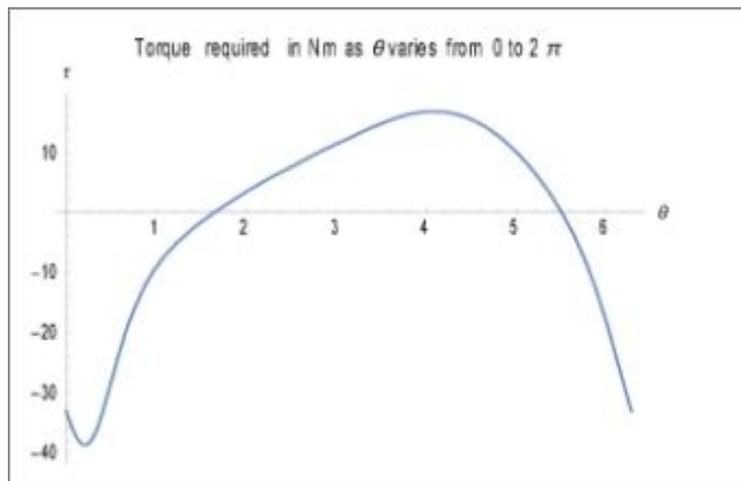


Figure 16: Torque required in mN mm as θ varies from 0 to 2π

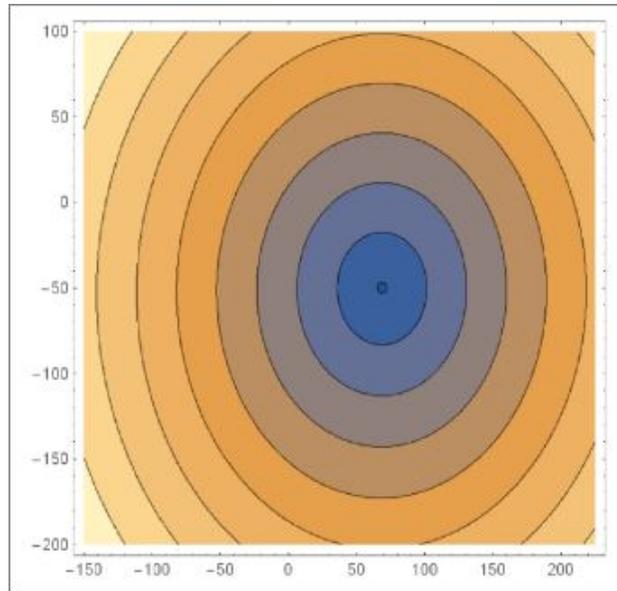


Figure 17: Contour plot of the maximum torque required for x_m varying from -155 to 225 and y_m varying from -200 to 100 (lengths in mm). The torque in the blue region at the center is 16.8015 N and in the pale peripheral region is 16.8017 N.

6.2 Finite Element Simulation

1. A finite element analysis was performed on a model of polypropylene shaft of the same dimensions of the original prototype to ensure that the shaft can take the loads.
2. It has to be noted that the shaft should be able to take a normal load of 20 N at the end passing through the bearings.
3. A maximum stress of 6.085 MPa is observed for the applied loads and encastre constraints which is safe compared to the ultimate tensile strength of 40 MPa.

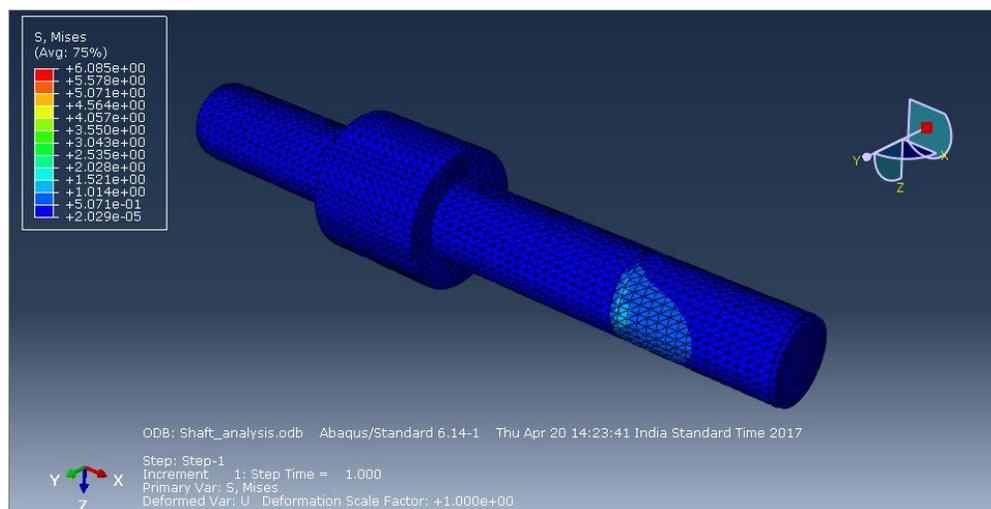


Figure 18: FEA simulation of the polypropylene shaft

7 Critical Analysis

The fixture of the four bar should be jutting out of the front face of the bin to use the coupler curve effectively, else the coupler curve is partially obscured by the bottom of the bin.

The tolerances involved in the design should be very accurate to run both the four bars precisely at the same time, else there would be a mismatch in the speeds of them and would result in their locking due to bending of their connecting shaft.

The sliders on which the weights are mounted on might not be able to function smoothly due to the moment of the weight. So a suitable rotary joint might be much suitable.

Both weights must be in perfect synchronization to perform the cleaning effectively and to preserve the structure of the cleaning arm.

Since the modules are exposed to very abusive environments there is a predominant vulnerability of the joints getting jammed so they must be sealed from the environment.

A small compliant end should be introduced to accommodate traversal over uneven surfaces.

8 References

1. Central Pollution Control Board, "Consolidated Annual report on implementation of municipal solid waste", April 2016
2. Swachh Bharat Mission, "Municipal Solid Waste Management Manual 1", May, 2016
3. Swachh Bharat Mission, "Municipal Solid Waste Management Manual 2", May, 2016

9 Appendix A - Mathematica Code

```
SetDirectory[NotebookDirectory[]]
<< mathutils_2220.m
<< fourbarutils_2220.m

"I:\PD Lab 2"

linkprops = {h -> 6, b -> 32,  $\rho$  -> 7.85/109};(*all lengths in mm*)

linklengths = {l10 -> 165, l11 -> 85, l12 -> 175, l13 -> 125, xc -> - 95,
  yc -> 375};

(* $\theta$  is the angle made by l10 with the vertical, will have to be \
adjusted if link lengths changed -- path should be aligned correctly to \
ground *)

coupler = couplerCurve[{0, 0},
  165 {-Sin[ $\theta$ ], Cos[ $\theta$ ]}, {85, 175, 125}, {-95, 375}, {x,
  y}] /. linklengths /.  $\theta$  -> 20 Degree;

 $\theta$  = 20 Degree;

soln = fourBarFK[{0, 0},
  165 {-Sin[ $\theta$ ], Cos[ $\theta$ ]}, {85, 175,
  125}, { $\theta$ 1,  $\theta$ 2,  $\theta$ 3}, br];

Manipulate[ (*code written by teaching assistants for ED2220 in Jul-Nov 2016 \
at IITM -- Aravind Baskar, M. Teja Krishna -- used with slight modification*)
\

soln = fourBarFK[b1, b2, {l1, l2, l3}, { $\theta$ 1,  $\phi$ 2,  $\phi$ 3}, br];
p1 = b1 + l1*{Cos[ $\theta$ 1], Sin[ $\theta$ 1]};
pcg = p1 + rot2D[ $\phi$ 2].pc /. soln;

(* Graphics for four-bar *)
fourbar =
fourBarPlotter[b1, b2, {l1, l2, l3},
  pc, { $\theta$ 1,  $\phi$ 2,  $\phi$ 3} /. soln];

(* Axes of coupler *)
axes = Graphics[{Arrow[{p1,
  p1 + (l2/2)*{Cos[ $\phi$ 2], Sin[ $\phi$ 2]} /. soln}],
  Arrow[{p1,
  p1 + (l2/2)*{Cos[ $\phi$ 2 + \[Pi]/2], Sin[ $\phi$ 2 + \[Pi]/2]} /. soln]}}];
```

```

crossHair =
Graphics[{Thickness[0.001],
  Line[{pcg - 0.5*{Cos[φ2], Sin[φ2]} /. soln,
    pcg + 0.5*{Cos[φ2], Sin[φ2]} /. soln}],
  Line[{pcg - 0.5*{Cos[φ2 + \[Pi]/2], Sin[φ2 + \[Pi]/2]} /. soln,
    pcg + 0.5*{Cos[φ2 + \[Pi]/2], Sin[φ2 + \[Pi]/2]} /. soln}],
  Orange, PointSize[0.01], Point[pcg]}}];

(* Coupler curve of four-bar *)
curvePlot = couplerNumeric[b1, b2, {l1, l2, l3}, pc];

(* Show the plots together *)
boundingBox = Graphics[{FaceForm[], Rectangle[{-15, -10}, {15, 15}]}];
Show[{ boundingBox, curvePlot, fourbar, axes, crossHair}, ImageSize -> Large]

, {{b1, {0, 0}}, Locator}, {{b2, 165 {-Sin[θ], Cos[θ]}},
Locator}, Style["Link lengths", Bold, 12], {{l1, 85}, 0.0, 85}, {{l2, 175},
0.0, 175}, {{l3, 125}, 0.0, 125}, {{θ1, \[Pi]/6}, 0,
2 \[Pi]}, Delimiter,
Style["Coupler Point (xc,yc)", Bold, 12], {{pc, {-95, 375}, ""}, 0, 10,
ControlType -> Slider2D, Appearance -> "Labeled"},
Style["Branch", Bold, 12], {{br, 2, ""}, {1, 2}}, ControlPlacement -> Left
]

θ = 20 Degree; (*fixed angle*)

\[Eta]1 = 11 Cos[θ1] + 12 Cos[θ2] - 13 Cos[θ3] -
10 Cos[θ];

\[Eta]2 = 11 Sin[θ1] + 12 Sin[θ2] - 13 Sin[θ3] -
10 Sin[θ];

elim[t1_, t2_, \[Beta]_] :=
Module[{matrix, exp},
matrix = Solve[{t1, t2} == {0, 0}, {Cos[\[Beta]], Sin[\[Beta]]}];
exp = (Cos[\[Beta]])^2 + (Sin[\[Beta]])^2 - 1 /. matrix;
TrigExpand[Simplify[Numerator[Together[exp]]]];
(*function to eliminate an angle \[Beta] from two equations involving its \
sine and cosine, with matrix and exp as local variables*)

trigSolve[t1_, \[Beta]_] :=
Module[{a, b, c}, a = Coefficient[t1, Cos[\[Beta]]];
b = Coefficient[t1, Sin[\[Beta]]];
c = t1 - a*Cos[\[Beta]] - b*Sin[\[Beta]]; \[Psi]possible =
TrigExpand[
Simplify[Together[{{ArcCos[-c/(a^2 + b^2)^0.5] +
ArcTan[a, b], -ArcCos[-c/(a^2 + b^2)^0.5] +
ArcTan[a,
b]]}]]; (*function to solve for angle \[Beta] from an equation t1 \
of the form aCos\[Beta] + bSin\[Beta] + c = 0*)

fk3 = trigSolve[elim[\[Eta]1, \[Eta]2, θ2], θ3] /. linklengths;

Plot[fk3, {θ1, 0, 2 \[Pi]}]

fk2 = trigSolve[elim[\[Eta]1, \[Eta]2, θ3], θ2] /. linklengths;

Show[%47, AxesLabel -> {HoldForm[θ1], HoldForm[θ2]},

```

```

PlotLabel ->
HoldForm[Variation of  $\theta_2$  as  $\theta_1$  varies from 0 to  $2 \pi$ ].
The discontinuity is due to addition of  $2 \pi$  and is not real],
LabelStyle -> {GrayLevel[0]}

p1 = l1 {-Sin[ $\theta_1$ ], Cos[ $\theta_1$ ]} /. linklengths /.  $\theta_2$  ->
fk2[[2]] /.  $\theta_3$  -> fk3[[2]];

p2 = 10 {-Sin[ $\theta$ ], Cos[ $\theta$ ]} +
13 {-Sin[ $\theta_3$ ], Cos[ $\theta_3$ ]} /. linklengths /.  $\theta_2$  ->
fk2[[2]] /.  $\theta_3$  -> fk3[[2]];

cp = p1 + ((rot2D[ $\pi/2 + \theta_2$ ].{xc, yc}) /.  $\theta_2$  -> fk2[[2]]) /.
linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  -> fk3[[2]];

pcm = {xm, ym} /. linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  ->
fk3[[2]];

(*mass centres*)

mc1 = p1/2 /. linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  -> fk3[[2]];

mc2 = (p2 + p1)/2 /. linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  ->
fk3[[2]];

mc3 = (p2 + 10 {-Sin[ $\theta$ ], Cos[ $\theta$ ]} )/2 /.
linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  -> fk3[[2]];

mc4 = (p1 + pcm)/2 /. linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  ->
fk3[[2]];

mc5 = {pcm[[1]] + cp[[1]][[1]], pcm[[2]] + cp[[2]][[1]]}/2 /.
linklengths /.  $\theta_2$  -> fk2[[2]] /.  $\theta_3$  -> fk3[[2]];

g = 9810 ; (*acc due to gravity, mm/s^2*)

- F[Delta]xc - g (m1 \[Delta]h1 + m2 \[Delta]h2 + m3 \[Delta]h3 + m4 \[Delta]h4 + m5 \[Delta]h5)
+ M\[Delta] $\theta_1$  = 0
M = F(\[PartialD]xc/\[PartialD] $\theta_1$ ) + g (m1 \[PartialD]h1/\[PartialD] $\theta_1$  + m2
\[PartialD]h2/\[PartialD] $\theta_1$  + m3 \[PartialD]h3/\[PartialD] $\theta_1$  + m4 \[PartialD]h4/\[PartialD] $\theta_1$ 
+ m5 \[PartialD]h5/\[PartialD] $\theta_1$ ) where all hi's are functions of  $\theta_1$ 
= F(\[PartialD]xc/\[PartialD] $\theta_1$ ) + g  $\rho$  b h (l1 \[PartialD]h1/\[PartialD] $\theta_1$  + l2
\[PartialD]h2/\[PartialD] $\theta_1$  + l3 \[PartialD]h3/\[PartialD] $\theta_1$  + l4 \[PartialD]h4/\[PartialD] $\theta_1$ 
+ l5 \[PartialD]h5/\[PartialD] $\theta_1$ )

F = 75000; (*estimate of force in mN needed to push garbage, slightly \
elevated estimate*)

l4 = ((p1[[1]] - pcm[[1]])^2 + (p1[[2]] - pcm[[2]])^2)^(1/2);

l5 = ((cp[[1]] - pcm[[1]])^2 + (cp[[2]] - pcm[[2]])^2)^(1/2);

Torque = (F *D[cp[[1]],  $\theta_1$ )] [
1]] + (g (( $\rho$  b h) /. linkprops) (l1 D[mc1[[2]],  $\theta_1$ ] +
12 D[mc2[[2]],  $\theta_1$ ][[1]] +
13 D[mc3[[2]],  $\theta_1$ ][[1]] + (l4 /. xm -> -95 /. ym -> 0) D[
mc4[[2]],  $\theta_1$ ] + (l5 /. xm -> -95 /. ym -> 0) [
D[mc5[[2]],  $\theta_1$ ][[1]]) /. linklengths /. xm -> -95 /.
ym -> 0)[[1]] ;

```

```

Plot[Torque/10^6, {θ1, 0, 2 \[Pi]}]

Show[%45, AxesLabel -> {HoldForm[θ], HoldForm[\[Tau]]},
PlotLabel ->
HoldForm[Torque required in Nm as θ varies from 0 to 2 \[Pi]],
LabelStyle -> {GrayLevel[0]}]

MaxValue[Torque, θ1] (*is in mN mm *)

1.68011*10^7

MinValue[Torque, θ1] (*is in mN mm *)

-3.87746*10^7

15 D[mc5[[2]], θ1][[1]]

{1/2 \[Sqrt][(-xm -
375 Cos[ArcCos[(
0.00285714 (-49450 +
28050 Cos[20 \[Degree] - θ1]))/(34450 -
28050 Cos[20 \[Degree] - θ1])^0.5] -
ArcTan[-28875 Cos[20 \[Degree]] +
14875 Cos[θ1], -28875 Sin[20 \[Degree]] +
14875 Sin[θ1]]] - 85 Sin[θ1] -
95 Sin[ArcCos[(
0.00285714 (-49450 +
28050 Cos[20 \[Degree] - θ1]))/(34450 -
28050 Cos[20 \[Degree] - θ1])^0.5] -
ArcTan[-28875 Cos[20 \[Degree]] +
14875 Cos[θ1], -28875 Sin[20 \[Degree]] +
14875 Sin[θ1]]])^2 + (-ym + 85 Cos[θ1] -
95 Cos[ArcCos[(
0.00285714 (-49450 +
28050 Cos[20 \[Degree] - θ1]))/(34450 -
28050 Cos[20 \[Degree] - θ1])^0.5] -
ArcTan[-28875 Cos[20 \[Degree]] +
14875 Cos[θ1], -28875 Sin[20 \[Degree]] +
14875 Sin[θ1]]] +
375 Sin[ArcCos[(
0.00285714 (-49450 +
28050 Cos[20 \[Degree] - θ1]))/(34450 -
28050 Cos[20 \[Degree] - θ1])^0.5] -
ArcTan[-28875 Cos[20 \[Degree]] +
14875 Cos[θ1], -28875 Sin[20 \[Degree]] +
14875 Sin[θ1]]])^2)}

Torquegen = (F *D[cp[[1]], θ1][[
1]] + (g ((ρ b h ) /. linkprops) (11 D[mc1[[2]], θ1] +
12 D[mc2[[2]], θ1][[1]] + 13 D[mc3[[2]], θ1][[1]] +
14 D[mc4[[2]], θ1] + 15 D[mc5[[2]], θ1][[1]]) /.
linklengths)[[1]] ;

14 D[mc4[[2]], θ1]

-(85/2) Sqrt[(-ym + 85 Cos[θ1])^2 + (-xm - 85 Sin[θ1])^2]
Sin[θ1]

```

```

ContourPlot[
Log[MaxValue[Torquegen,  $\theta_1$ ]], {xm, -500, 500}, {ym, -500, 500}]

AbortedContourPlot[MaxValue[Torquegen,
\theta1], xm, -150, 225, ym, -200, 100]MaxValue[Torquegen/.xm- > -50/.ym- > 70,
\theta1]1.68016 * 107Minimize[MaxValue[Torquegen,
\theta1], xm, ym]General :: stop : FurtheroutputofNMaximize :: nnumwillbesuppressedduringthiscalculation. >>
Aborted

```

The location of xm, ym is not making much difference to the torque plot and its peak. Hence the most time and cost efficient design has been chosen i.e. two perpendicular links.
